

Unit -4

Cams

Cam:-

A cam is a mechanical member for transmitting a desired motion to a follower by direct contact.

(or)

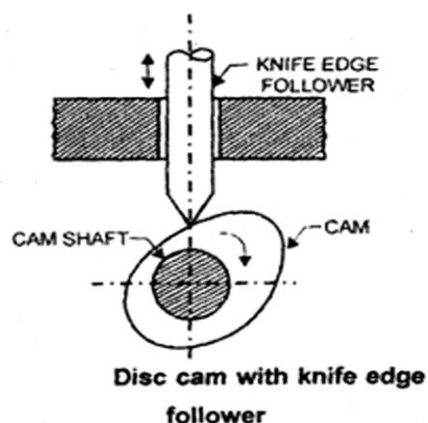
A cam is a rotating machine element which gives reciprocating or oscillating motion to another element known as follower.

The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at uniform speed by a shaft, but the follower motion is predetermined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modern machinery today. The cams are widely used for operating the engines, automatic attachment of machineries, paper cutting machines, and automatic lathes machineries....etc

Types of followers:-

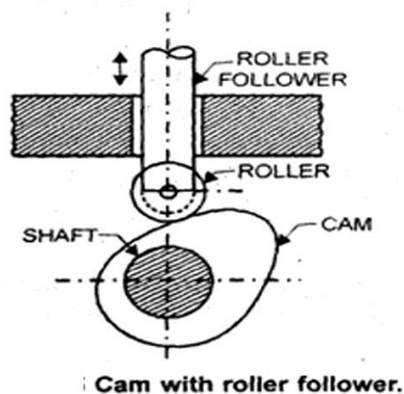
1. according to the surface in contact:-

(a) knife edge follower:- when the rotating end of the follower, as shown in figure. The sliding motion takes place between the contacting surface (i.e., the knife edge and the cam surface). It is seldom used in practice because the small area of contacting surface results in excessive wear. In knife edge followers, a considerable thrust exists between the follower and the guide.



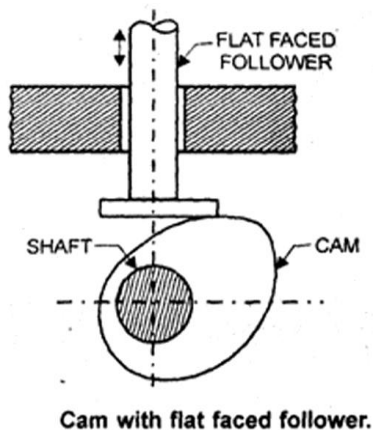
(b) roller follower :-

When the contacting end of the follower is a roller, it is called a roller follower, as shown in figure. Since the rolling motion takes place between the contacting surfaces (i.e., the roller and the cam), therefore the rate of wear is greatly reduced. In roller followers also the side thrust exists between the follower and the guide. The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircrafts engines.



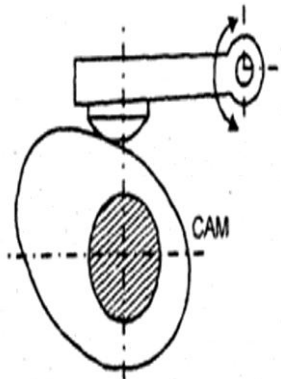
(c) flat faced (or) mushroom follower:-

When the contacting end of the follower is a perfectly flat face, it is called flat faced follower, as shown in figure. It may be noted that the side thrust between the follower and the guide is much reduced in case of flat faced followers. Flat faced follower causes high surface stress to minimize these stresses used the spherical faced follower.



(d) spherical faced follower :-

When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in figure. It may be noted that when a flat faced follower is used in automobile engines, high surface stresses are produced . in order to minimize these stresses, the flat end of the follower is machined to a spherical shape.



Cam with spherical faced oscillating follower

2. according to othe motion of the follower:-

The follower according to its motion, are of the following two types,

(a) Reciprocating (or) translating follower :-

When the follower reciprocates in guides as the cam rotates uniformly, it is called reciprocates or translating follower. The follower as shown in fig (a to d) all reciprocating or translating followers.

(b) oscillating or rotating follower :-

When the uniform rotaty motion of the cam is connerted into predetermined oscillatory motion of the follower, it is called oscillatory or rotating follower. The follower, as shown in fig, is an oscillatory or rotating follower. Fig (d)is the oscillating follower.

3. according to the path of of motion of the follower :-

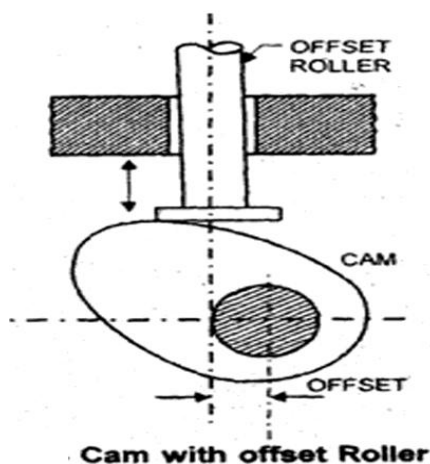
These are two types,

(a) Radial follower:-

When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower. The followers, as shown in fig (a to d) are radial followers.

(b) off - set follower:-

When the motion of the follower is along an axis away from the axis of the cam centre, it is called off-set follower. The follower, as shown in fig.

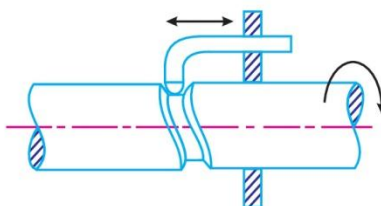


Types of cams :-

Cams are classified into two types. They are,

(1) Radial or disc cam :-

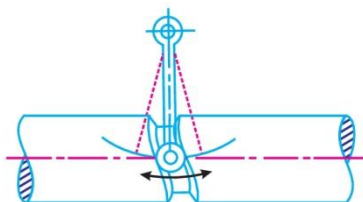
In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in fig.



Cylindrical cam with reciprocating follower.

(2) cylindrical cam :-

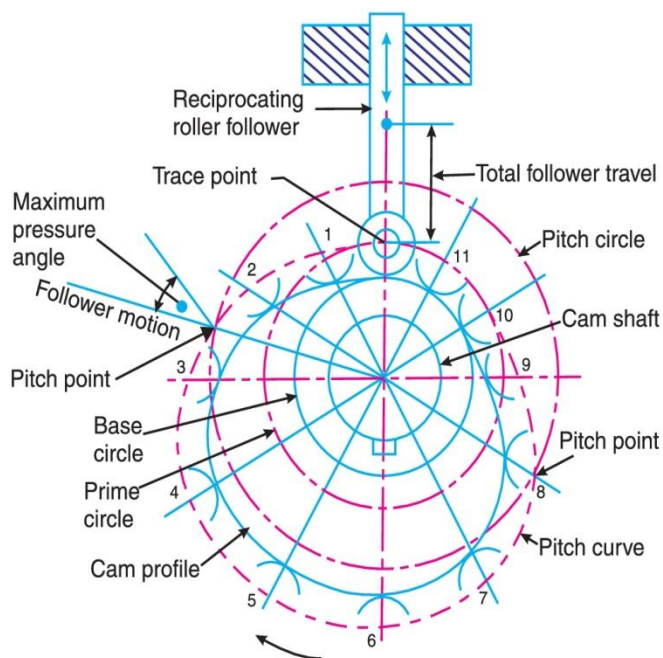
In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower shown in fig.



Cylindrical cam with oscillating follower.

Terminology use in cams :-

Below fig shows a radial cam with reciprocating roller follower. The following terms are important in order to draw the cam profile.



Terms used in radial cams.

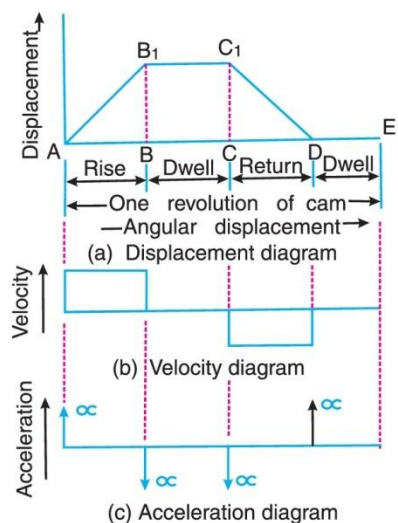
1. Base circle:- it is the smallest circle that can be drawn to the cam profile.
2. Trace point :- The trace point is used to generate the pitch curve. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the trace point.
3. pressure angle :- it is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.
4. pitch point :- it is a point on the pitch curve having the maximum pressure angle.
5. pitch circle :- it is a circle drawn from the centre of the cam through the pitch points.
6. pitch curve :- it is the curve generated by trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same where as for a roller follower, they are separated by the radius of the roller.
7. prime circle :- it is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve.
8. lift or stroke :- it is the maximum travel of the follower from its lowest position to the top most position.
9. angle of Dwell :- it is the angle through which the cam turns while the follower remains stationary at the highest or the lowest position.
10. angle of action :- it is the total angle moved by the cam during the time between the beginning of rise and the end of the return of the follower.

Types of follower motions :-

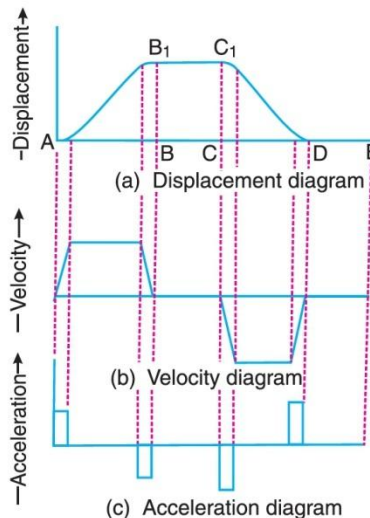
The different types of followers motions are as follows,

1. uniform velocity
2. Simple harmonic motion
3. uniform acceleration and retardation, and

4. Cycloidal.

1. uniform velocity :-

Displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.



Modified displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

as the follower moves with uniform velocity during its rise and return stroke, therefore the displacement curves should have a constant slope. The displacement, velocity and acceleration diagrams drawn by taking time on x-axis and displacement, velocity or acceleration respectively on y-axis. These figures are shown below. Since the follower moves with uniform velocity, therefore the lines AB and CD should be straight. The acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is mainly due to the reason that the follower has to start from rest and has to gain velocity within no time.

2. simple harmonic motion :-

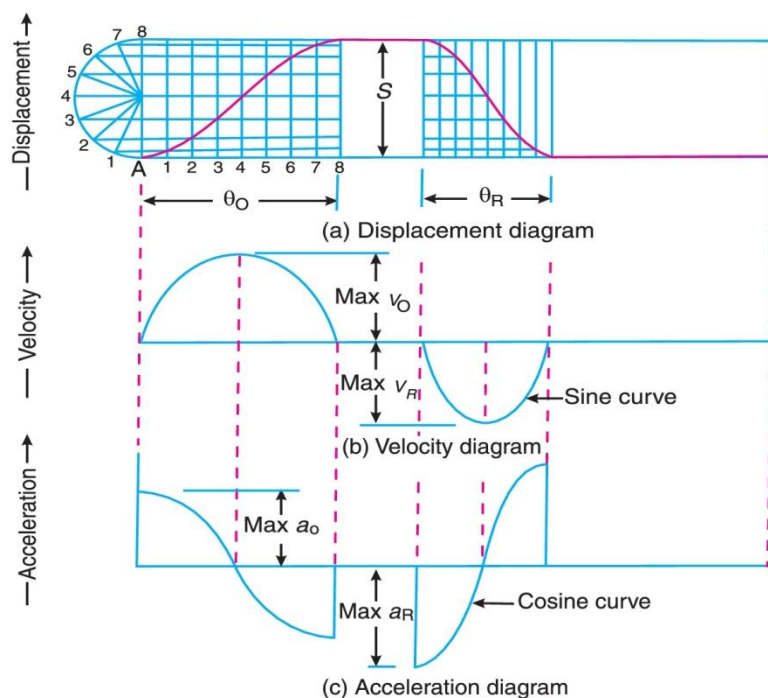
The displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion are shown in figures. The displacement diagram is drawn as follows.

- draw a semi-circle on the follower stroke as diameter.
- divide the semi-circle into any number of even equal parts (say eight).

- c. divide the angular displacements of the cam during out stroke and return stroke into the same number of equal parts.
- d. the displacement diagram is obtained by projecting the points as shown in fig.

The velocity and acceleration diagrams are shown in figures. Since the follower moves with a simple harmonic motion, therefore velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve.

We see from fig. that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke on the other hand, the acceleration of the follower is maximum at the beginning and at the ends of the stroke and diminishes to zero at mid-stroke.



Displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion.

Let

S= stroke of the follower

θ_0 and θ_L = angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians.

ω =angular velocity of the cam in rad/ sec.

Maximum acceleration of the follower on the out stroke

$$a_0 = a_p = \frac{\pi^2 \omega^2 .s}{2(\theta_0)^2}$$

Maximum velocity of the follower on the return stroke

$$V_R = \frac{\pi\omega .s}{2 \theta_R}$$

Maximum acceleration of the follower on the return stroke

$$a_R = \frac{\pi^2 \omega^2 .s}{2(\theta_R)^2}$$

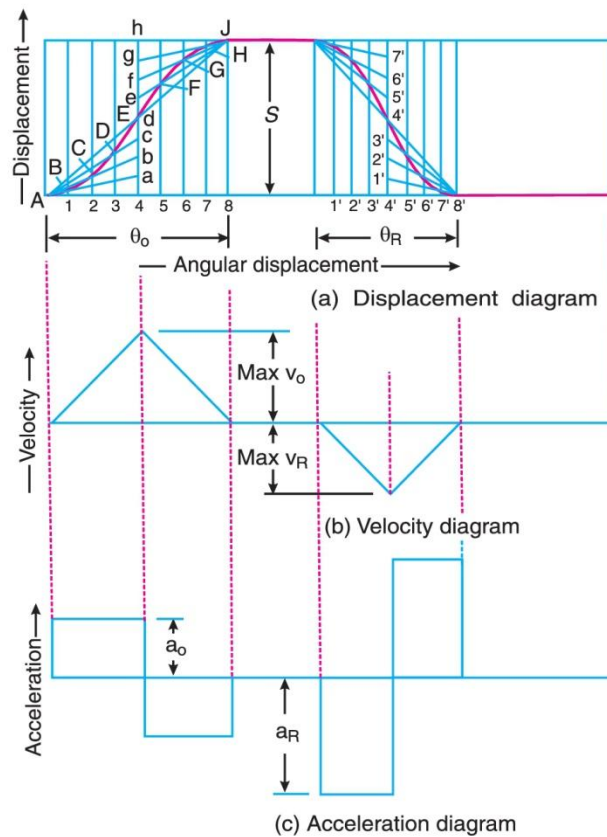
Maximum velocity of the follower on the outstroke

$$V_o = \frac{\pi\omega .s}{2 \theta_o}$$

3. uniform acceleration and retardation :-

The displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation are shown in fig. we see that the displacement diagram consists of a parabolic curve and may be drawn as discussed below:

- divide the angular displacement of the cam during outstroke into any even number of equal parts. And drawn vertical lines through these points as shown in figure.
- divide the stroke of the follower into the same number of equal even parts.
- join Aa to intersect the vertical line through point 1 at B. similarly, obtain the other points C, D ..etc. now join these points to obtain the parabolic curve for the outstroke of the follower.
- in the similar way as discussed above, the displacement diagram for the follower during return stroke may be drawn.



Displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation.

Maximum velocity of the follower on the outstroke

$$V_o = \frac{2\omega \cdot s}{\theta_o}$$

Maximum velocity of the follower on the return stroke

$$V_R = \frac{2\omega \cdot s}{\theta_R}$$

Maximum acceleration of the follower during out stroke

$$a_o = \frac{4 \omega^2 \cdot s}{(\theta_o)^2}$$

Maximum acceleration of the follower on the return stroke

$$a_R = \frac{4 \omega^2 \cdot s}{(\theta_R)^2}$$

4.cycloidal motion :-

It is out of the syllabus.

Construction of cam profile for a radial cam :-

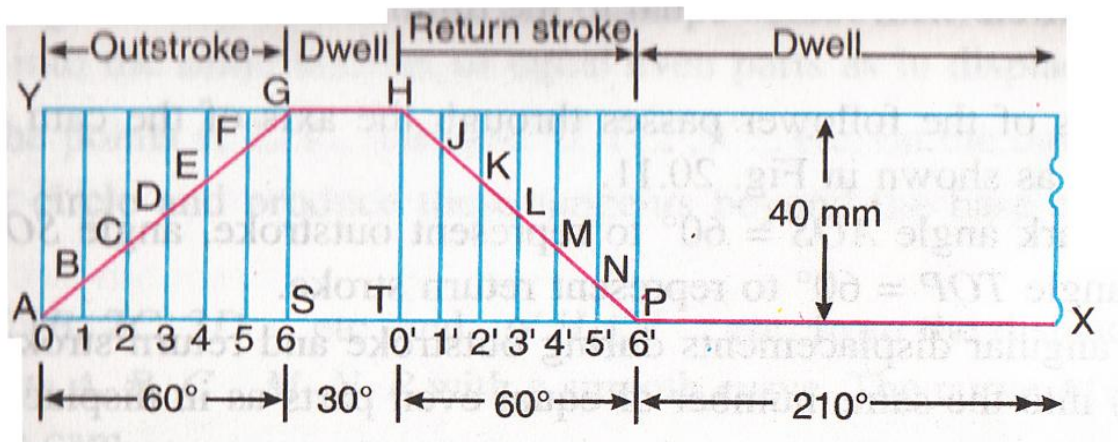
1. in order to draw the cam profile for a radial cam, first of all the displacement diagram. For the given motion of the follower is drawn.
2. the follower must rotate in the opposite direction to the cam rotation.
3. then by constructing the follower unit proper positions at each angular position the profile of the working surface of the cam is drawn.

Problems on these 3 follower motions :-

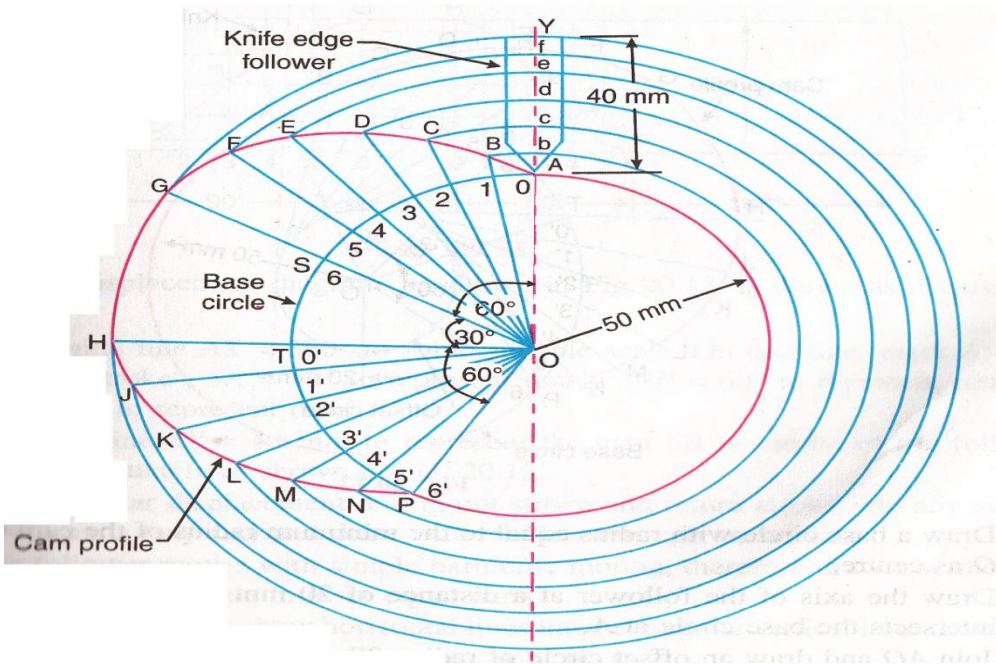
1. a cam is to give the following motion to a knife -edged follower:

1. outstroke during 60 degree of cam rotation.
2. dwell for the next 30 degree of cam rotation.
3. return stroke during next 60 degree of cam rotation and
4. dwell for the remaining 210 degree of cam rotation.

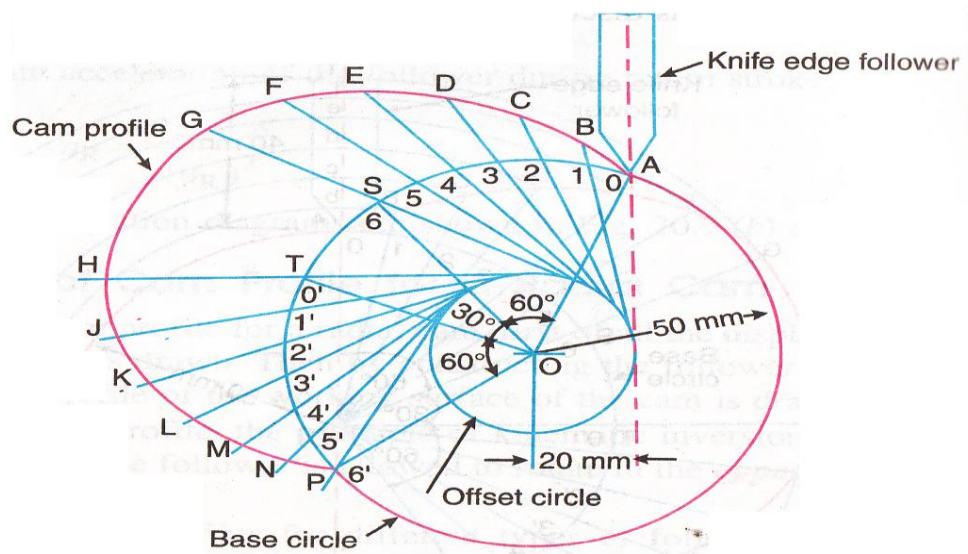
The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. the follower moves with uniform velocity during both the outstroke and return stroke. Draw the profile of the cam when (a) the axis of the follower passes through the axis of the cam shaft, and (b) the axis of the follower is offset by 20 mm from axis of the cam shaft.

Displacement diagram :-

(a):-



(b):-



Example 20.2. A cam is to be designed for a knife edge follower with the following data :

1. Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.
2. Dwell for the next 30° .
3. During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion.
4. Dwell during the remaining 180° .

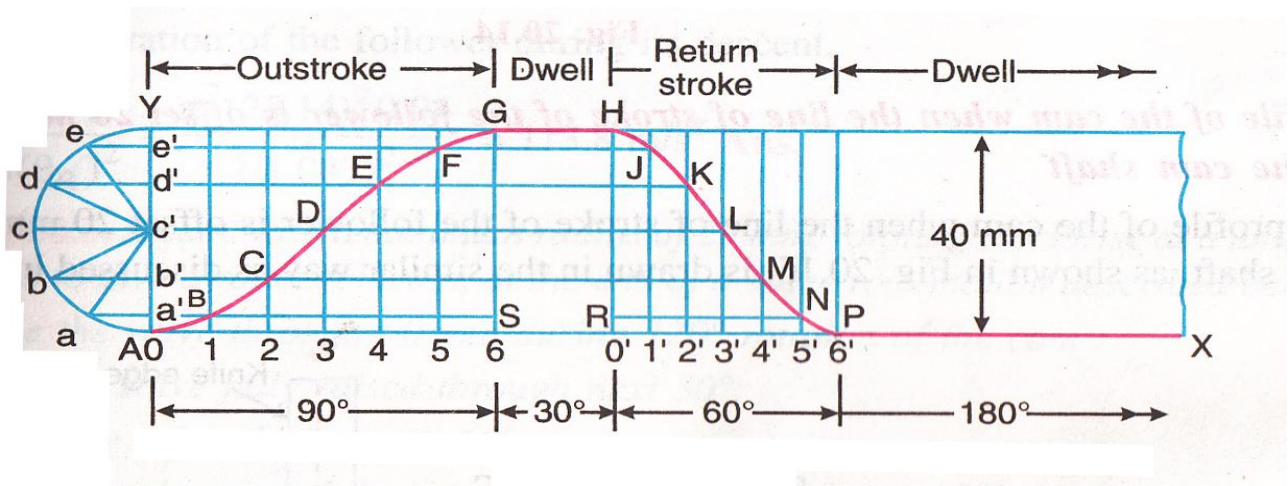
Draw the profile of the cam when

(a) the line of stroke of the follower passes through the axis of the cam shaft, and

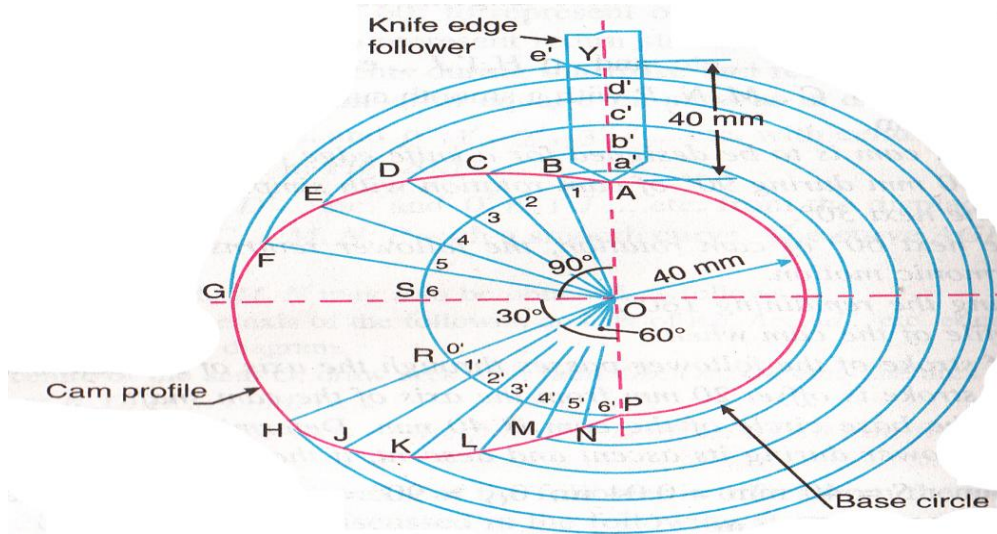
(b) the line of stroke is offset 20 mm from the axis of the cam shaft.

The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

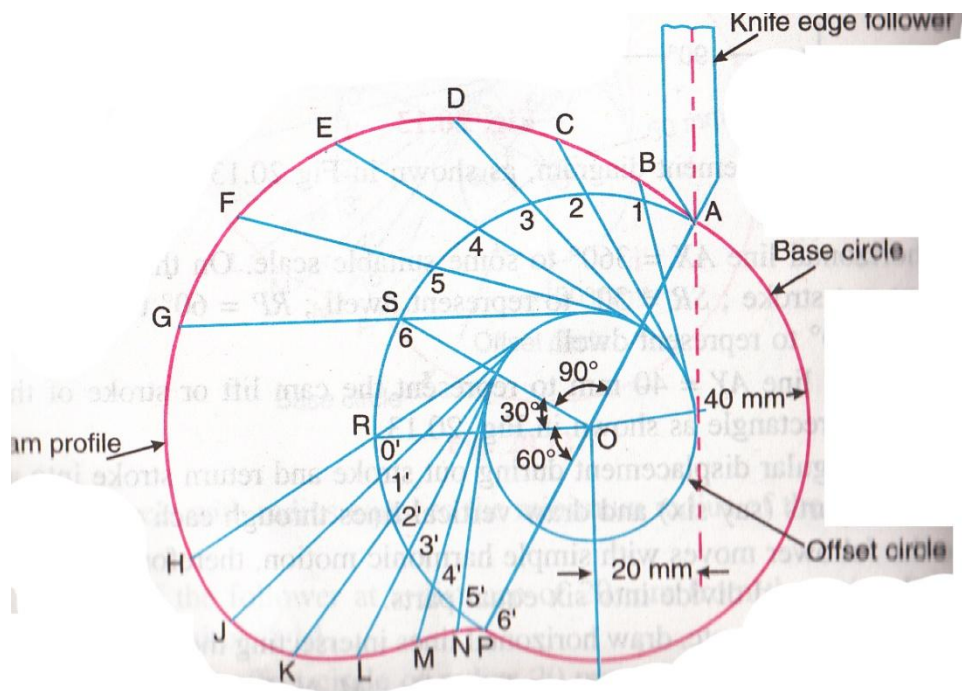
Displacement diagram:-



(a):-



(b):-



Maximum velocity of the follower during its ascent and descent

We know that angular velocity of the cam,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14 \text{ rad/s}$$

We also know that the maximum velocity of the follower during its ascent,

$$v_O = \frac{\pi \omega . S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s Ans.}$$

and maximum velocity of the follower during its descent,

$$v_R = \frac{\pi \omega . S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s Ans.}$$

Maximum acceleration of the follower during its ascent and descent

We know that the maximum acceleration of the follower during its ascent,

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.571)^2} = 50.6 \text{ m/s}^2 \text{ Ans.}$$

and maximum acceleration of the follower during its descent,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.047)^2} = 113.8 \text{ m/s}^2 \text{ Ans.}$$

Example 20.6. A cam, with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to give a knife edge follower the motion as described below :

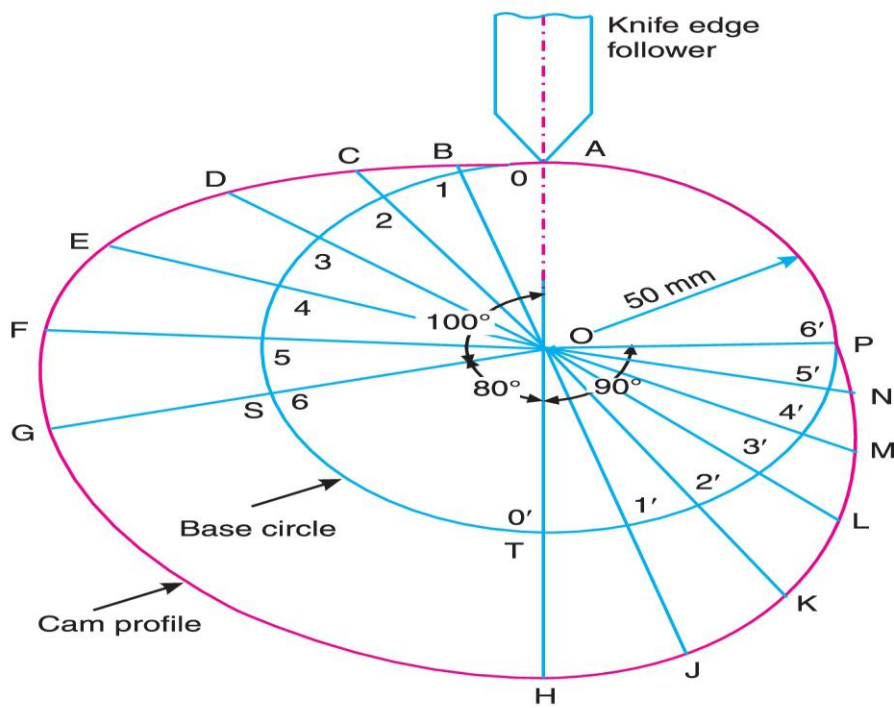
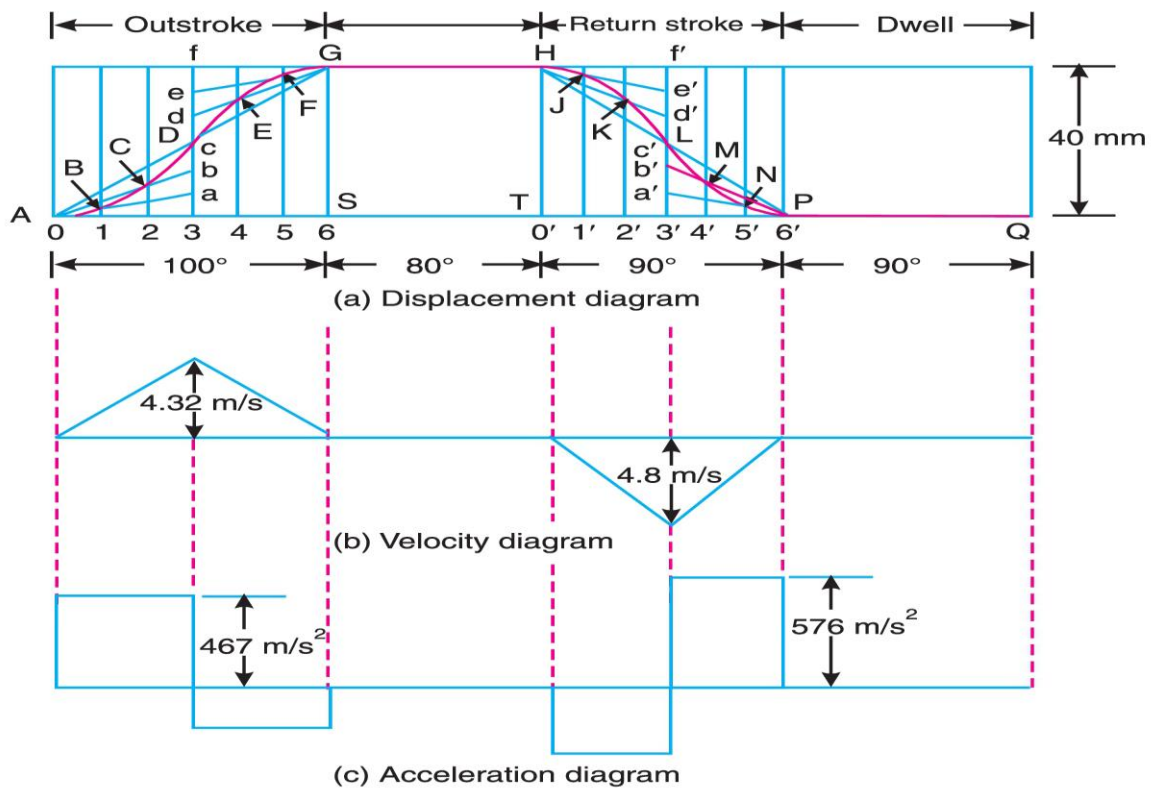
1. To move outwards through 40 mm during 100° rotation of the cam ; **2.** To dwell for next 80° ; **3.** To return to its starting position during next 90° , and **4.** To dwell for the rest period of a revolution i.e. 90° .

Draw the profile of the cam

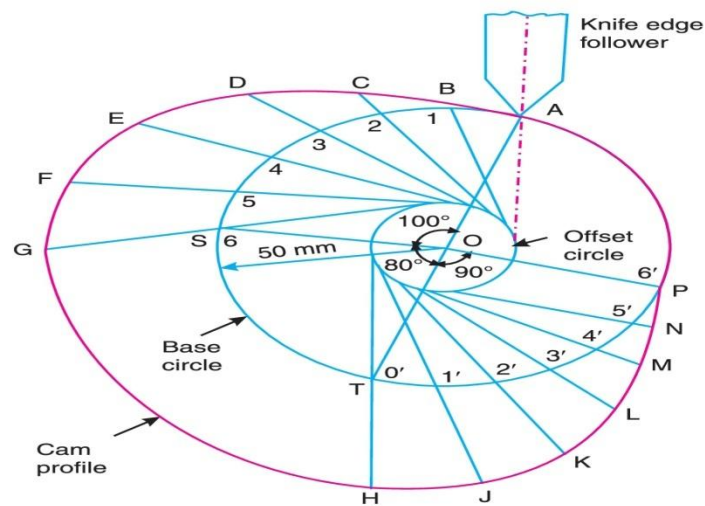
- (i) when the line of stroke of the follower passes through the centre of the cam shaft, and
- (ii) when the line of stroke of the follower is off-set by 15 mm.

The displacement of the follower is to take place with uniform acceleration and uniform retardation. Determine the maximum velocity and acceleration of the follower when the cam shaft rotates at 900 r.p.m.

Draw the displacement, velocity and acceleration diagrams for one complete revolution of the cam.



(b):-



Maximum velocity of the follower during out stroke and return stroke

We know that angular velocity of the cam shaft,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

We also know that the maximum velocity of the follower during out stroke,

$$v_O = \frac{2\omega S}{\theta_O} = \frac{2 \times 94.26 \times 0.04}{1.745} = 4.32 \text{ m/s Ans.}$$

and maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega S}{\theta_R} = \frac{2 \times 94.26 \times 0.04}{1.571} = 4.8 \text{ m/s Ans.}$$

The velocity diagram is shown in Fig. 20.24 (b).

Maximum acceleration of the follower during out stroke and return stroke

We know that the maximum acceleration of the follower during out stroke,

$$a_O = \frac{4\omega^2 S}{(\theta_O)^2} = \frac{4(94.26)^2 \times 0.04}{(1.745)^2} = 467 \text{ m/s}^2 \text{ Ans.}$$

and maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\omega^2 S}{(\theta_R)^2} = \frac{4(94.26)^2 \times 0.04}{(1.571)^2} = 576 \text{ m/s}^2 \text{ Ans.}$$

The acceleration diagram is shown in Fig. 20.24 (c).

Analysis of motion of followers:-**Tangent Cam with Reciprocating Roller Follower**

When the flanks of the cam are straight and tangential to the base circle and nose circle, then the cam is known as a **tangent cam**, as shown in Fig. 20.40. These cams are usually symmetrical about the centre line of the cam shaft. Such type of cams are used for operating the inlet and exhaust valves of internal combustion engines. We shall now derive the expressions for displacement, velocity and acceleration of the follower for the following two cases :

1. When the roller has contact with the straight flanks ; and
2. When the roller has contact with the nose.

Let r_1 = Radius of the base circle or minimum radius of the cam,
 r_2 = Radius of the roller,
 r_3 = Radius of nose,
 α = Semi-angle of action of cam or angle of ascent,
 θ = Angle turned by the cam from the beginning of the roller displacement,
 ϕ = Angle turned by the cam for contact of roller with the straight flank, and
 ω = Angular velocity of the cam.

1. When the roller has contact with straight flanks. A roller having contact with straight flanks is shown in Fig. 20.40. The point O is the centre of cam shaft and the point K is the centre of nose. EG and PQ are straight flanks of the cam. When the roller is in lowest position, (*i.e.* when the roller has contact with the straight flank at E), the centre of roller lies at B on the pitch curve. Let the cam has turned through an angle* θ (less than ϕ) for the roller to have contact at any point (say F) between the straight flanks EG . The centre of roller at this stage lies at C . Therefore displacement (or lift or stroke) of the roller from its lowest position is given by

$$x = OC - OB = \frac{OB}{\cos \theta} - OB = OB \left(\frac{1 - \cos \theta}{\cos \theta} \right)$$

$$= (r_1 + r_2) \left(\frac{1 - \cos \theta}{\cos \theta} \right) \quad \dots (\because OB = OE + EB = r_1 + r_2) \quad \dots (i)$$

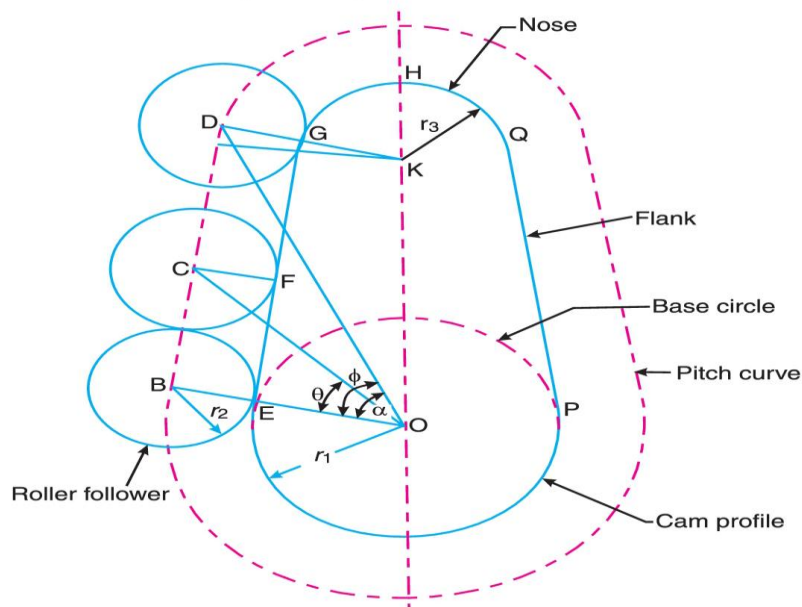


Fig. 20.40. Tangent cam with reciprocating roller follower having contact with straight flanks.

Differentiating equation (i) with respect to t , we have velocity of the follower,

$$\begin{aligned} v &= \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = (r_1 + r_2) \left(\frac{\sin \theta}{\cos^2 \theta} \right) \frac{d\theta}{dt} \\ &= \omega(r_1 + r_2) \left(\frac{\sin \theta}{\cos^2 \theta} \right) \quad \dots (\because d\theta/dt = \omega) \dots \text{(ii)} \end{aligned}$$

From equation (ii), we see that when θ increases, $\sin \theta$ increases and $\cos \theta$ decreases. In other words, $\sin \theta / \cos^2 \theta$ increases. Thus the velocity is maximum where θ is maximum. This happens when $\theta = \phi$ i.e. when the roller just leaves contact with the straight flank at G or when the straight flank merges into a circular nose.

\therefore Maximum velocity of the follower,

$$v_{max} = \omega(r_1 + r_2) \left(\frac{\sin \phi}{\cos^2 \phi} \right)$$

Now differentiating equation (ii) with respect to t , we have acceleration of the follower,

$$\begin{aligned} a &= \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} \\ &= \omega(r_1 + r_2) \left(\frac{\cos^2 \theta \cdot \cos \theta - \sin \theta \times 2 \cos \theta \times -\sin \theta}{\cos^4 \theta} \right) \frac{d\theta}{dt} \\ &= \omega^2 (r_1 + r_2) \left(\frac{\cos^2 \theta + 2 \sin^2 \theta}{\cos^3 \theta} \right) \quad \dots \left(\because \frac{d\theta}{dt} = \omega \right) \\ &= \omega^2 (r_1 + r_2) \left[\frac{\cos^2 \theta + 2(1 - \cos^2 \theta)}{\cos^3 \theta} \right] \\ &= \omega^2 (r_1 + r_2) \left(\frac{2 - \cos^2 \theta}{\cos^3 \theta} \right) \quad \dots \text{(iii)} \end{aligned}$$

A little consideration will show that the acceleration is minimum when $\frac{2 - \cos^2 \theta}{\cos^3 \theta}$ is minimum. This is only possible when $(2 - \cos^2 \theta)$ is minimum and $\cos^3 \theta$ is maximum. This happens

$$a = \omega^2 \cdot r \left[\cos \theta_1 + \frac{L^2 \cdot r \cos 2\theta_1 + r^3 \sin^4 \theta_1}{(L^2 - r^2 \sin^2 \theta_1)^{3/2}} \right]$$

In a symmetrical tangent cam operating a roller follower, the least radius of the cam is 30 mm and roller radius is 17.5 mm. The angle of ascent is 75° and the total lift is 17.5 mm. The speed of the cam shaft is 600 r.p.m. Calculate : 1. the principal dimensions of the cam ; 2. the accelerations of the follower at the beginning of the lift, where straight flank merges into the circular nose and at the apex of the circular nose. Assume that there is no dwell between ascent and descent.

Solution. Given : $r_1 = 30$ mm ; $r_2 = 17.5$ mm ;
 $\alpha = 75^\circ$; Total lift = 17.5 mm ; $N = 600$ r.p.m. or
 $\omega = 2\pi \times 600/60 = 62.84$ rad/s

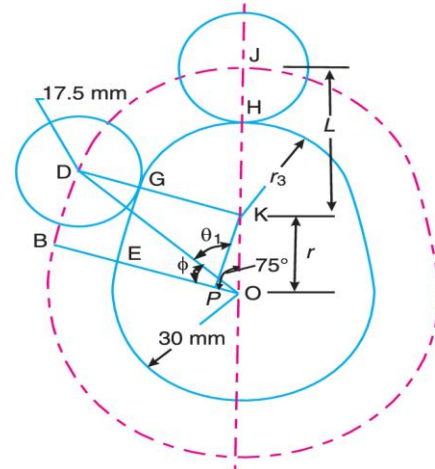


Fig. 20.42

1. Principal dimensions of the cam

Let $r = OK$ = Distance between cam centre and nose centre,
 r_3 = Nose radius, and
 ϕ = Angle of contact of cam with straight flanks.

From the geometry of Fig. 20.42,

$$\begin{aligned} r + r_3 &= r_1 + \text{Total lift} \\ &= 30 + 17.5 = 47.5 \text{ mm} \end{aligned}$$

$$\therefore r = 47.5 - r_3 \quad \dots (i)$$

Also, $OE = OP + PE$ or $r_1 = OP + r_3$

$$\therefore OP = r_1 - r_3 = 30 - r_3 \quad \dots (ii)$$

Now from right angled triangle OKP ,

$$OP = OK \times \cos \alpha \quad \dots (\because \cos \alpha = OP/OK)$$

or $30 - r_3 = (47.5 - r_3) \cos 75^\circ = (47.5 - r_3) 0.2588 = 12.3 - 0.2588 r_3$

$$\dots (\because OK = r)$$

$$\therefore r_3 = 23.88 \text{ mm Ans.}$$

and $r = OK = 47.5 - r_3 = 47.5 - 23.88 = 23.62 \text{ mm Ans.}$

Again, from right angled triangle ODB ,

$$\tan \phi = \frac{DB}{OB} = \frac{KP}{OB} = \frac{OK \sin \alpha}{r_1 + r_2} = \frac{23.62 \sin 75^\circ}{30 + 17.5} = 0.4803$$

$$\therefore \phi = 25.6^\circ \text{ Ans.}$$

2. Acceleration of the follower at the beginning of the lift

We know that acceleration of the follower at the beginning of the lift, *i.e.* when the roller has contact at E on the straight flank,

$$\begin{aligned} a_{min} &= \omega^2 (r_1 + r_2) = (62.84)^2 (30 + 17.5) = 187\,600 \text{ mm/s}^2 \\ &= 187.6 \text{ m/s}^2 \text{ Ans.} \end{aligned}$$

Acceleration of the follower where straight flank merges into a circular nose

We know that acceleration of the follower where straight flank merges into a circular nose *i.e.* when the roller just leaves contact at G ,

$$\begin{aligned} a_{max} &= \omega^2 (r_1 + r_2) \left[\frac{2 - \cos^2 \phi}{\cos^3 \phi} \right] = (62.84)^2 (30 + 17.5) \left(\frac{2 - \cos^2 25.6^\circ}{\cos^3 25.6^\circ} \right) \\ &= 187\,600 \left(\frac{2 - 0.813}{0.733} \right) = 303\,800 \text{ mm/s}^2 = \mathbf{303.8 \text{ m/s}^2 \text{ Ans.}} \end{aligned}$$

Acceleration of the follower at the apex of the circular nose

We know that acceleration of the follower for contact with the circular nose,

$$a = \omega^2 . r \left[\cos \theta_1 + \frac{L^2 . r \cos 2\theta_1 + r^3 \sin^4 \theta_1}{(L^2 - r^2 \sin^2 \theta_1)^{3/2}} \right]$$

Since θ_1 is measured from the top position of the follower, therefore for the follower to have contact at the apex of the circular nose (*i.e.* at point H), $\theta_1 = 0$.

\therefore Acceleration of the follower at the apex of the circular nose,

$$\begin{aligned} a &= \omega^2 . r \left(1 + \frac{L^2 . r}{L^3} \right) = \omega^2 . r \left(1 + \frac{r}{L} \right) = \omega^2 . r \left(1 + \frac{r}{r_2 + r_3} \right) \\ &= (62.84)^2 23.62 \left(1 + \frac{23.62}{17.5 + 23.88} \right) = 146\,530 \text{ mm/s}^2 \quad \dots (\because L = r_2 + r_3) \\ &= 146.53 \text{ m/s}^2 \text{ Ans.} \end{aligned}$$